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Unsteady MHD free convective flow of a compressible fluid past a moving vertical plate in the presence of radiative heat transfer

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Abstract

The paper studies the free convection flow of a compressible Boussinesq fluid under the simultaneous action of buoyancy and transverse magnetic field while the Rosselant approximation has been invoked to describe the radiative flux in the energy equation. The viscosity of the fluid v and its thermal conductivity k in this model are assumed to be functions of temperature. Under suitable non-dimensionalization the governing non-linear, coupled, partial differential equations are solved employing a perturbation technique based on the assumption that the fluid flow field is made up of a steady part and a transient. Results obtained which compare favourably well with published data show, that the skin friction for a compressible fluid is lower than that for an incompressible fluid.
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1. Introduction

The study of free convection flow past a vertical plate has applications in many areas of science and engineering. Quite a few studies have been done in this area looking at different aspects of applications where the vertical plate sometimes is immersed in a porous medium but only a few of these works will be mentioned here. Singh and Dikshit [4] studied hydrodynamic flow past a continuously moving semi-infinite plate with large suction, while in Bestman [11] the focus was on chemically reacting species. As noted in Bestman and Adjepong [1,2], in the presence of radiative heat we cannot regard the viscosity v and the thermal conductivity k as constant since these in general are known to vary with temperature. Having said that, Ogulu and Bestman [7,8] in earlier studies have shown that for blood flow studies, it is safe to assume constant viscosity (Newtonian) for blood within the realms of physiotherapy despite the fact that some of the blood vessels involved have diameters comparable with the radius of the red blood cells. Raptis [10] studied the flow of a micro-polar fluid past a continuously moving plate by the presence of radiation assuming a constant viscosity. Takhar et al. [9] also studied radiation effects on MHD free convection flow of a fluid past a semi-infinite vertical plate where the viscosity v and the thermal conductivity k where assumed constant.

Azzam [5] studied radiation effects on the MHD mixed free-forced convective flow past a semi-infinite moving vertical plate for high temperature differences where though the viscosity v and the thermal conductivity k were assumed to vary with temperature, the fluid was not regarded as a Boussinesq fluid hence the motivation for this study which involves the effect of radiation on unsteady free convection flow of a compressible Boussinesq fluid past a semi-infinite vertical plate. We propose to study the effect of radiation on free convection flow of a compressible Boussinesq fluid since most of the studies mentioned above assumed the fluid to be incompressible. The procedure adopted here is as outlined below.

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Nomenclature

u	velocity components
y	coordinate
T	dimensional temperature
k	thermal conductivity
c_p	specific heat at constant pressure
β	coefficients of volume expansion due to temperature
μ	permeability
Pr	Prandtl number
σ^*	Stefan–Boltzmann constant
V_0	scale of free stream velocity
ρ	fluid density
t	time
ρ	density
ε	time corrective parameter, ($\varepsilon \ll 1$)
M	magnetic Hartmann number
k^*	Rosseland absorption coefficient
θ	non-dimensional temperature
g	acceleration due to gravity
σ	electrical conductivity
H_0	magnetic field
q	radiative flux vector
G_r	free convection parameter
v	kinematic coefficient of viscosity
U_p	plate velocity

Superscript

$'$	differentiation with respect to z

Subscripts

w	wall condition
∞	free stream condition

2. Formulation of the problem

We consider the unsteady two-dimensional flow of an electrically conducting fluid. As in Bestman and Adjepong [1,2], we assume that the fluid viscosity v varies as the temperature T in the form

$$v \propto T^{\omega}, \tag{1a}$$

where ω is a constant, (for water $\omega = 0.76$). The flow is buoyancy driven and the fluid obeys the simple Boussinesq equation of state

$$\rho_\infty - \rho = \rho_\infty \beta (T - T_\infty). \tag{1b}$$

We further assume that induced magnetic fields are negligible, there is no external electric field present and flow velocities are slow hence no viscous dissipation heating. The plate is infinite therefore all variables are functions of y and t only except the pressure. With these assumptions and those usually associated with the Boussinesq approximation the proposed governing equations for this model (see Fig. 1) are

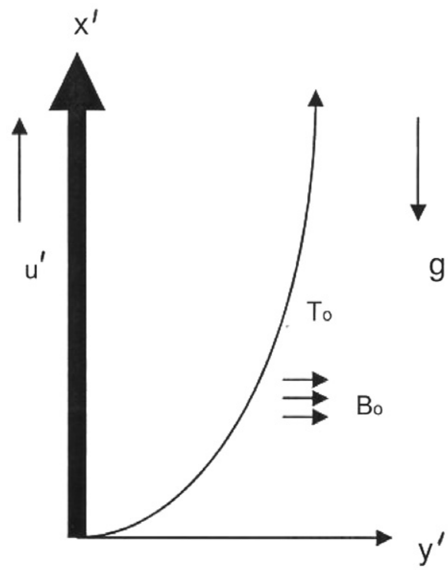

Fig. 1. The physical co-ordinate system.

$$\frac{\partial v'}{\partial y'} = 0 \tag{2}$$

$$\left(\frac{\partial u'}{\partial t'} + v'\frac{\partial u'}{\partial y'}\right) = -\frac{1}{\rho}\frac{\partial p'}{\partial x'} + v\frac{\partial^2 u'}{\partial y'^2} - \frac{\mu^2 \sigma H_0'^2 u'}{\rho} + g\beta(T - T_\infty) \tag{3}$$

$$\rho c_p \left(\frac{\partial T}{\partial t'} + v'\frac{\partial T}{\partial y'}\right) = \frac{\partial}{\partial y'}\left(k\frac{\partial T}{\partial y'}\right) - \frac{\partial q'}{\partial y'} \tag{4}$$

Subject to the boundary conditions

$$\begin{aligned} &u' = U_0, \quad T = T_w \quad \text{at } y' = 0, \\ &u' \to U_\infty, \quad T \to T_\infty \quad \text{as } y' \to \infty. \end{aligned} \tag{5a, b}$$

Though v and k vary with temperature, the fluid still obeys the Boussinesq equation, Bestman and Adjepong [1]. Furthermore, from Eq. (2) we take the constant suction normal to the plate and write

$$v' = -V_0 \tag{6}$$

Outside the boundary layer Eq. (3) gives

$$-\frac{1}{\rho}\frac{dp'}{dx'} = \frac{dU'_\infty}{dt'} + \frac{\sigma\mu^2 H_0^2 U'_\infty}{\rho}. \tag{7}$$

For the energy equation we invoke the Rosseland approximation for the radiative flux thus

$$\frac{\partial q'}{\partial y'} = -\frac{4\sigma^*}{3k^*}\frac{\partial T^4}{\partial y'} \tag{8}$$

In view of Eqs. (6)–(8) our leading equations become

$$\frac{\partial u}{\partial t}=\frac{dU_\infty}{dt}+\frac{\partial}{\partial y}\left(\theta^\omega\frac{\partial\theta}{\partial y}\right)+\frac{\partial u}{\partial y}-M^2(u-U_\infty)+Gr\theta \tag{9}$$

$$\frac{\partial\theta}{\partial t}=\frac{1}{Pr}\frac{\partial}{\partial y}\left(\theta^\omega\frac{\partial\theta}{\partial y}\right)+R_a\frac{\partial^2\theta}{\partial y^2} \tag{10}$$

The appropriate boundary conditions now are

$$\begin{aligned}&\theta=1,\quad u=U_p\quad \text{at } y=0\\&\theta\to 0,\quad u\to U_\infty\quad \text{as } y\to\infty.\end{aligned} \tag{11}$$

In Eqs. (9) and (10) we have introduced the following non-dimensional variables

$$\begin{aligned}&u'=uU_0,\quad u_p=U_pU_0,\quad v'=vV_0,\quad y'=\frac{vy}{V_0},\\&U'_\infty=U_0U_\infty,\quad t'=\frac{tv}{V_0^2},\quad \theta=\frac{T-T_\infty}{T_w-T_\infty},\quad M=\frac{\sigma H_0^2}{\rho V_0^2},\\&Pr=\frac{v\rho c_p}{k},\quad R_a=\frac{16\sigma^* T_\infty^3}{3kk^*},\quad Gr=\frac{g\beta v}{V_0^3}(T-T_\infty)\end{aligned} \tag{12}$$

The problem in mathematical terms involves the solution of Eqs. (9) and (10) subject to the boundary conditions in (11). The problem depends on the free convection parameter or Grashoff number Gr, Prandtl number Pr, the square of the Hartman number M^2 and the radiation parameterR_a.

3. Method of solution

The problem as posed in Eqs. (9) and (10) is non-linear and coupled, however if we assume the Prandtl number is constant then in the spirit of Bestman and Adjepong [1,2] we can seek asymptotic expansions for our flow variables about a small parameter ε (a time corrective parameter), thereby splitting the problem into two components – a steady state and a transient state; thus

$$\begin{aligned}&u(y,t)=u^{(0)}(y)+\varepsilon u^{(1)}(y,t)+\cdots\\&\theta(y,t)=\theta^{(0)}(y)+\varepsilon\theta^{(1)}(y,t)+\cdots\end{aligned} \tag{13}$$

When we substitute Eq. (13) into Eqs. (9) and (10) we obtain the following sequence of approximations

$$\frac{1}{Pr}\frac{d}{dy}\left(\theta^{(0)}\frac{d\theta^{(0)}}{dy}\right)+R_a\frac{d^2\theta^{(0)}}{dy^2}=0 \tag{14}$$

$$\frac{d\theta^{(0)\omega}}{dy}+\frac{du^{(0)}}{dy}+M^2u^{(0)}+Gr\theta^{(0)}=0 \tag{15}$$

$$\left.\begin{aligned}&u^{(0)}=U_p,\quad \theta^{(0)}=\theta_w\quad \text{at } y=0\\&u^{(0)}=1,\quad \theta^{(0)}\to 0\quad \text{as } y\to\infty\end{aligned}\right\}t>0, \tag{16}$$

where θ_w is the wall normalized temperature,

$$\frac{\partial\theta^{(1)}}{\partial t}=\frac{1}{Pr}\frac{\partial^2}{\partial y^2}(\theta^{(0)\omega}\theta^{(1)})+R_a\frac{\partial^2\theta^{(1)}}{\partial y^2} \tag{17}$$

$$\begin{aligned}\frac{\partial u}{\partial t}=&\frac{\partial}{\partial y}\left(\theta^{(0)\omega}\frac{\partial u^{(1)}}{\partial y}\right)+\omega\frac{\partial}{\partial y}\left(\theta^{(0)\omega-1}\frac{du^{(0)}}{dy}\theta^{(1)}\right)\\&-M^2u^{(1)}+Gr\theta^{(1)}\end{aligned} \tag{18}$$

$$\left.\begin{aligned}&u^{(1)}=0,\quad \theta^{(1)}=1,\quad \text{at } y=0\\&u^{(1)}=0,\quad \theta\to 0,\quad \text{as } y\to\infty\end{aligned}\right\}t>0. \tag{19}$$

We continue the analysis by re-casting Eq. (14) as

$$\frac{d}{dy}\left(\frac{d\theta^{(0)\omega+1}}{dy}\right)+\frac{d}{dy}\left((\omega+1)PrR_a\frac{d\theta^{(0)}}{dy}\right)=0. \tag{20}$$

Integrating Eq. (20) twice subject to the conditions in Eq. (16), we can show that

$$\theta^{(0)}(y)=\theta_w. \tag{21}$$

From Eq. (21) we have, $\frac{d\theta^{(0)\omega}}{dy}=0$, hence Eq. (15) now becomes

$$\frac{du^{(0)}}{dy}+M^2(u^{(0)}-1)-Gr\theta_w=0 \tag{22}$$

which can be solved subject to the conditions in Eq. (16) to obtain

$$u^{(0)}(y)=1+(U_p-1)e^{-M^2y}+\frac{Gr\theta_w}{M^2}. \tag{23}$$

Next we cast Eq. (17) in the form

$$\frac{\partial^2\theta^{(1)}}{\partial y^2}=k^2\frac{\partial\theta^{(1)}}{\partial t} \tag{24}$$

where, $k^2=\frac{Pr}{\theta^{(0)\omega}+PrR_a}$. Employing Laplace transforms we can show from Eq. (24) subject to appropriate boundary conditions in Eq. (19) that

$$\theta^{(1)}(y,t)=\operatorname{erfc}\left(\frac{ky}{2\sqrt{t}}\right). \tag{25}$$

On appeal to Abramowitz and Stegun [6], we can cast Eq. (25) in the form

$$\theta^{(1)}(y,t)=\frac{2}{\pi}\int_{\frac{ky}{2\sqrt{t}}}^{\infty}e^{-v^2}dv\quad t>0. \tag{26}$$

The solution of Eq. (18) is expedited by writing it in the form

$$\frac{\partial u^{(1)}}{\partial t}=\theta^{(0)\omega}\frac{\partial^2u^{(1)}}{\partial y^2}+\omega\theta^{(0)\omega}\frac{\partial}{\partial y}\left(\frac{\theta^{(1)}}{\theta^{(0)}}\frac{du^{(0)}}{dy}\right)-M^2u^{(1)}+Gr\theta^{(1)}. \tag{27}$$

On the strength of Eq. (26) we can write

$$\frac{\theta^{(1)}}{\theta^{(0)}}=\frac{\frac{2}{\pi}\int_{\frac{ky}{2\sqrt{t}}}^{\infty}e^{-v^2}dt}{\theta_w} \tag{28}$$

$$\frac{\theta^{(1)}}{\theta^{(0)}}\to 1 \text{ as } y\to 0\quad\text{and}\quad \frac{\theta^{(1)}}{\theta^{(0)}}\to 0 \text{ as } y\to\infty.$$

For t small, $\frac{\theta^{(1)}}{\theta^{(0)}}$ will be small since $\frac{ky}{2\sqrt{t}} \to \infty$ for small t. For t large the steady state case prevails, hence it is reasonable and convenient to assume $\frac{\theta^{(1)}}{\theta^{(0)}} = 0$, so that Eq. (27) reduces to

$$\frac{\partial u^{(1)}}{\partial t} = \theta^{(0)\omega}\frac{\partial^2 u^{(1)}}{\partial y^2} - M^2 u^{(1)} + Gr\theta^{(1)}. \tag{29}$$

Taking the Laplace transform of Eq. (29) subject to the boundary and initial conditions, Tokis [3]:

$$\begin{aligned} &u^{(1)} = U_0 f(t) \quad \text{at } y = 0,\\ &u^{(1)} \to 0 \quad \text{as } y \to \infty, \end{aligned} \tag{30}$$

we obtain

$$\theta^{(0)\omega}\frac{d^2\bar{u}^{(1)}}{dy^2} - (M^2 + s)\bar{u}^{(1)} = -\frac{Gr}{s}e^{-y\sqrt{ks}} \tag{31}$$

and the boundary conditions are now

$$\bar{u}^{(1)}(0, s) = U_0 f(s)\bar{u}^{(1)}(\infty, s) = 0. \tag{32}$$

Solution of Eq. (31) subject to (32) gives

$$\begin{aligned} \bar{u}^{(1)} = {} & U_0\bar{f}(s)e^{-\frac{y\sqrt{s+M^2}}{\sqrt{\theta^{(0)\omega}}}}\\ & + \frac{Gr\theta^{(0)\omega}}{s(M^2 - s(k\theta^{(0)\omega} - 1))}\left\{e^{-y\sqrt{ks}} - e^{-\frac{y\sqrt{s+M^2}}{\sqrt{\theta^{(0)\omega}}}}\right\}. \end{aligned} \tag{33}$$

Appealing to tables of inverse transforms in Abramowitzc and Stegun [6] Eq. (33) gives

$$\begin{aligned} &u^{(1)} = \phi(y,t)\int_0^t \phi_*(y,\tau)f(t-\tau) + A_i(y,t)\\ &\phi_* = \frac{U_0 H(t)}{2\sqrt{\pi}}\frac{y}{\sqrt{\theta^{(0)\omega}}}t^{3/2}e^{-M^2 t} - \frac{y^2}{4t\sqrt{\theta^{(0)\omega}}}. \end{aligned} \tag{34}$$

$H(t)$ is the heaviside step function and $A_i(y,t)$, $i = 1, 2$ is the sum of the inversion of the other terms on the right hand side of Eq. (33). We now consider three special cases of the function $f(t)$ in Eq. (30);

(i) Single pulse. In this special case $f(t) = H(t)$. Substituting this in Eq. (34) we can show that

$$\begin{aligned} u^{(1)}(y,t) = {} & \frac{H(t)}{2\theta^{(0)\omega}}\left\{e^{-yM}\text{erfc}\left(\frac{y}{2\sqrt{t}} - M\sqrt{t}\right)\right.\\ & \left. + e^{yM}\text{erfc}\left(\frac{y}{2\sqrt{t}} + M\sqrt{t}\right)\right\} + A_i(y,t). \end{aligned} \tag{35}$$

(ii) Accelerated motion. In this special case we put $f(t) = \frac{t}{t_0}H(t)$ where t_0 is a constant, Eq. (34) now gives

$$\begin{aligned} u^{(1)}(y,t) = {} & \frac{H(t)}{t_0}\left\{\left(t - \frac{y}{2M}\right)e^{-yM}\text{erfc}\left(\frac{y}{2\sqrt{t}} - M\sqrt{t}\right)\right\}\\ & + \frac{H(t)}{t_0}\left\{\left(t + \frac{y}{2M}\right)e^{yM}\text{erfc}\left(\frac{y}{2\sqrt{t}} + M\sqrt{t}\right)\right\}\\ & + A_i(y,t) \end{aligned} \tag{36}$$

(iii) Decaying oscillatory motion.

Here $f(t) = Re\left[H(t)e^{-(\lambda^2 - i\zeta)t}\right] = \frac{H(t)}{2}\left[e^{-(\lambda^2 - i\zeta)t} + e^{-(\lambda^2 + i\zeta)t}\right]$, where λ and $\zeta\,(>0)$ are real dimensionless constants. Substituting for $f(t)$ now in Eq. (34) gives

$$\begin{aligned} u^{(1)}(y,t) = {} & \frac{H(t)}{4}e^{-(\lambda^2 - i\zeta)t}\left\{e^{-y(\alpha_1 + i\beta_1)}\text{erfc}\left(\frac{y - (\alpha_1 + i\beta)t}{2\sqrt{t}}\right)\right.\\ & \left. + e^{y(\alpha_1 + i\beta_1)}\text{erfc}\left(\frac{y + (\alpha_1 + i\beta_1)}{2\sqrt{t}}\right)\right\}\\ & + \frac{H(t)}{4}e^{-(\lambda^2 + i\zeta)t}\left\{e^{-y(\alpha_2 - i\beta_2)}\text{erfc}\left(\frac{y - (\alpha_2 - i\beta_2)t}{2\sqrt{t}}\right)\right.\\ & \left. + e^{y(\alpha_2 + i\beta_2)}\text{erfc}\left(\frac{y + (\alpha_2 - i\beta_2)t}{2\sqrt{t}}\right)\right\} + A_i(y,t) \end{aligned} \tag{37}$$

where

$$\alpha_1 + i\beta_1 \equiv \left[M - \lambda^2 + i\zeta\right]^{1/2} \quad \alpha_2 - i\beta_2 \equiv \left[M - \lambda^2 - i\zeta\right]^{1/2}.$$

It is now convenient to write out the expressions for $A_i(y,t)$. When $k\theta^{(0)\omega} \neq 1$ and

$$b = \frac{M^2}{(k\theta^{(0)\omega} - 1)}$$

we have,

$$\begin{aligned} A_1(y,t) = {} & \frac{Gr\theta^{(0)\omega}}{M^2}\text{erfc}\left(\frac{y}{2}\sqrt{\frac{k}{t}}\right)\\ & - \frac{Gr\theta^{(0)\omega}}{2M^2\sqrt{\theta^{(0)\omega}}}e^{bt}\left\{e^{-yM}\text{erfc}\left(\frac{y}{2\sqrt{\theta^{(0)\omega}t}} - M\sqrt{bt}\right)\right.\\ & \left. + e^{yM}\text{erfc}\left(\frac{y}{2\sqrt{\theta^{(0)\omega}t}} + M\sqrt{t}\right)\right\}\\ & + \frac{Gr\theta^{(0)\omega}}{2M^2\sqrt{\theta^{(0)\omega}}}e^{bt}\left\{e^{-yM\sqrt{b}}\text{erfc}\left(\frac{y}{2\sqrt{\theta^{(0)\omega}t}} - M\sqrt{bt}\right)\right.\\ & \left. + e^{yM}\text{erfc}\left(\frac{y}{2\sqrt{\theta^{(0)\omega}t}} + M\sqrt{t}\right)\right\}\\ & - \frac{Gr\theta^{(0)\omega}}{2M^2\sqrt{\theta^{(0)\omega}}}e^{bt}\left\{e^{-y\sqrt{bk}}\text{erfc}\left(\frac{y}{2}\sqrt{\frac{k}{t}} - \sqrt{bt}\right)\right.\\ & \left. + e^{y\sqrt{bk}}\text{erfc}\left(\frac{y}{2}\sqrt{\frac{k}{t}} + \sqrt{bt}\right)\right\} \end{aligned} \tag{38}$$

and when $k\theta^{(0)\omega} = 1$ we have

$$\begin{aligned} A_2(y,t) = {} & \frac{Gr\theta^{(0)\omega}}{M^2}\text{erfc}\left(\frac{y}{2}\sqrt{\frac{k}{t}}\right)\\ & - \frac{Gr\theta^{(0)\omega}}{2M^2\sqrt{\theta^{(0)\omega}}}\left\{e^{-yM}\text{erfc}\left(\frac{y}{2\sqrt{\theta^{(0)\omega}t}} - M\sqrt{t}\right)\right.\\ & \left. + e^{yM}\text{erfc}\left(\frac{y}{2\sqrt{\theta^{(0)\omega}t}} + M\sqrt{t}\right)\right\}. \end{aligned} \tag{39}$$

Having obtained complete expressions for the transient velocity we can now obtain the skin friction. Defining the non-dimensional skin friction τ as

$$\tau = \left.\frac{\partial u^{(1)}}{\partial y}\right|_{y=0}$$

we will have;

(i) Single pulse.

$$\tau_1^{(1)} = \left.\frac{\partial u^{(1)}}{\partial y}\right|_{y=0} = -H(t)M\mathrm{erfc}(M\sqrt{t}) - \frac{H(t)}{\sqrt{\pi t}} + \frac{Gr\theta^{(0)\omega}}{M}\sqrt{\frac{k\theta^{(0)\omega}}{\left(k\theta^{(0)\omega}-1\right)}}\mathrm{e}^{bt}(\mathrm{erfc}(\sqrt{bt}) - \mathrm{erfc}(\sqrt{bkt})) + \frac{Gr\theta^{(0)\omega}}{M}\mathrm{erfc}(M\sqrt{t}), \tag{40}$$

when $k\theta^{(0)\omega} \neq 1$, and

$$\tau_2^{(1)} = \left.\frac{\partial u^{(1)}}{\partial y}\right|_{y=0} = -H(t)M\mathrm{erfc}(M\sqrt{t}) - \frac{H(t)}{\sqrt{\pi t}} + \frac{Gr\theta^{(0)\omega}}{M}\mathrm{erfc}(M\sqrt{t}) + \frac{Gr\theta^{(0)\omega}}{M^2\sqrt{\pi t}}(\mathrm{e}^{-M^2t} - 1), \tag{41}$$

when $k\theta^{(0)\omega} = 1$.

(ii) Accelerated motion.

$$\tau_1^{(2)} = \left.\frac{\partial u^{(1)}}{\partial y}\right|_{y=0} = -\frac{H(t)}{t_0}\left\{\left(\frac{1}{M} + Mt\right)\right\}\mathrm{erfc}(M\sqrt{t}) + \sqrt{\frac{t}{\pi}}\mathrm{e}^{-M^2t} + \frac{Gr\theta^{(0)\omega}}{M}\sqrt{\frac{k\theta^{(0)\omega}}{\left(k\theta^{(0)\omega}-1\right)}}\mathrm{e}^{bt} \times \left(\mathrm{erfc}(\sqrt{bt}) - \mathrm{erfc}(\sqrt{bkt})\right) + \frac{Gr\theta^{(0)\omega}}{M}\mathrm{erfc}(M\sqrt{t}), \tag{42}$$

when $k\theta^{(0)\omega} \neq 1$, and

$$\tau_2^{(2)} = \left.\frac{\partial u^{(1)}}{\partial y}\right|_{y=0} = -\frac{H(t)}{t_0}\left\{\left(\frac{1}{M} + Mt\right)\right\}\mathrm{erfc}(M\sqrt{t}) + \sqrt{\frac{t}{\pi}}\mathrm{e}^{-M^2t} + \frac{H(t)}{\sqrt{\pi t}} + \frac{Gr\theta^{(0)\omega}}{M}\mathrm{erfc}(M\sqrt{t}) + \frac{Gr\theta^{(0)\omega}}{M^2\sqrt{\pi t}}(\mathrm{e}^{-M^2t} - 1), \tag{43}$$

when $k\theta^{(0)\omega} = 1$.

(iii) Decaying oscillatory motion.

$$\tau_1^{(3)} = \left.\frac{\partial u^{(1)}}{\partial y}\right|_{y=0} = -\frac{H(t)}{2}\mathrm{e}^{-(\lambda^2 - \mathrm{i}\zeta)t}(\alpha + \mathrm{i}\beta)\mathrm{erfc}[(\alpha + \mathrm{i}\beta) - \sqrt{t}] - \frac{H(t)}{2}\mathrm{e}^{-(\lambda^2 + \mathrm{i}\zeta)t}(\alpha - \mathrm{i}\beta)\mathrm{erfc}[(\alpha - i\beta) - \sqrt{t}] - \frac{H(t)}{\sqrt{\pi t}}\mathrm{e}^{-M^2t} + \frac{Gr\theta^{(0)\omega}}{M}\sqrt{\frac{k\theta^{(0)\omega}}{\left(k\theta^{(0)\omega}-1\right)}}\mathrm{e}^{bt} \times \left(\mathrm{erfc}(\sqrt{bt}) - \mathrm{erfc}(\sqrt{bkt})\right) + \frac{Gr\theta^{(0)\omega}}{M}\mathrm{erfc}(M\sqrt{t}), \tag{44}$$

when $k\theta^{(0)\omega} \neq 1$, and

$$\tau_2^{(3)} = \left.\frac{\partial u^{(1)}}{\partial y}\right|_{y=0} = -\frac{H(t)}{2}\mathrm{e}^{-(\lambda^2 - \mathrm{i}\zeta)t}(\alpha + \mathrm{i}\beta)\mathrm{erfc}[(\alpha + \mathrm{i}\beta) - \sqrt{t}] - \frac{H(t)}{2}\mathrm{e}^{-(\lambda^2 + \mathrm{i}\zeta)t}(\alpha - \mathrm{i}\beta)\mathrm{erfc}[(\alpha - \mathrm{i}\beta) - \sqrt{t}] - \frac{H(t)}{\sqrt{\pi t}}(\mathrm{e}^{-M^2t} - 1) + \frac{Gr\theta^{(0)\omega}}{M}\sqrt{\frac{k\theta^{(0)\omega}}{\left(k\theta^{(0)\omega}-1\right)}}\mathrm{e}^{bt} \times (\mathrm{erfc}(\sqrt{bt}) - \mathrm{erfc}(\sqrt{bkt})) + \frac{Gr\theta^{(0)\omega}}{M}\mathrm{erfc}(M\sqrt{t}) + \frac{Gr\theta^{(0)\omega}}{M}\mathrm{erfc}(M\sqrt{t}) + \frac{Gr\theta^{(0)\omega}}{M^2\sqrt{\pi t}}(\mathrm{e}^{-M^2t} - 1), \tag{45}$$

when $k\theta^{(0)\omega} = 1$.

The solutions are now complete.

4. Results and discussion

An analysis is presented for the effects of radiation on the unsteady compressible flow of a Boussinesq fluid past a semi-infinite vertical flat plate. This problem is governed by a set of coupled, non-linear, partial differential equations (9)–(11) which are solved employing asymptotic expansions about a small geometric parameter ε ($\varepsilon \ll 1$), followed by integration with respect to y, the span-wise coordinate. Three special cases are considered in the analysis but only one of them, the single pulse case, will be discussed here for brevity. For the purpose of the numerical discussion we choose suitable values of the parameters of the problem to show the applicability of the solutions obtained. For the Prandtl number Pr, we choose $Pr = 7$, as corresponds to water or $Pr = 0.7$ as corresponds to air. The other parameters are varied arbitrarily. As noted in Mbeledogu and Ogulu [12], as the time parameter t, increases we approach steady state so $t = 0.15$ is within the transient problem. Fig. 2 shows the effect of the parameters of the problem on the temperature distribution where we observe that an increase in the radiation parameter

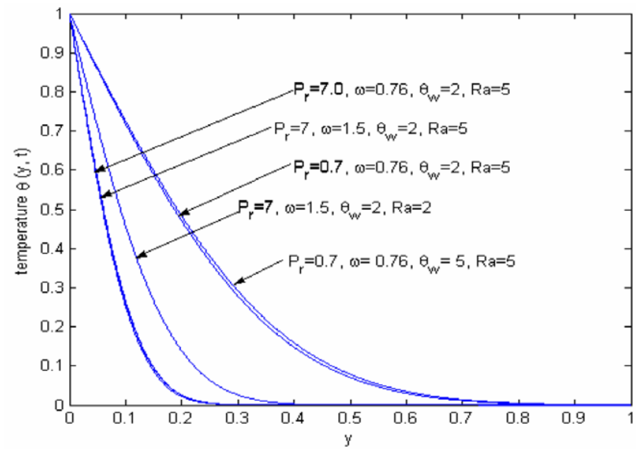

Fig. 2. Temperature distribution, $t = 0.15$.

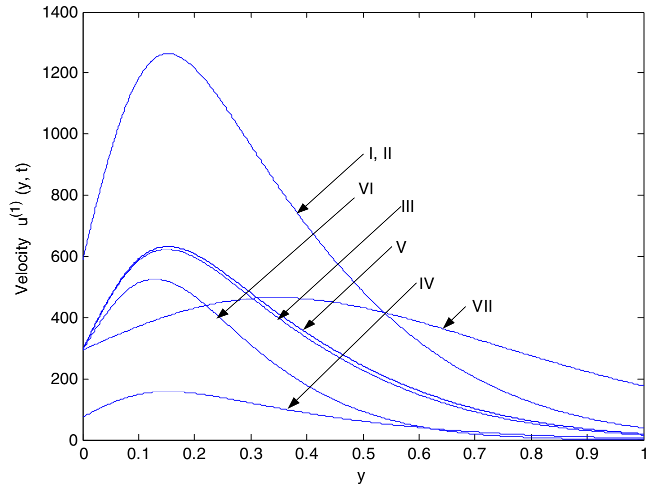

Fig. 3. Plot of $u^{(1)}$ versus y from Eq. (35).

leads to a decrease in the temperature boundary layer and hence the temperature, while increase in the Prandtl number leads to a decrease in the temperature boundary layer. Variation in the exponential index ω or the wall normalization temperature θ_w, have little effect on the temperature distribution.

Fig. 3 shows the effect of material parameters on the velocity distribution for a single pulse with $k\theta^{(0)} = 1$ and $H(t) = 1$. Fig. 3 shows that generally, the velocity distribution reaches a maximum value near the plate before it then decreases to the free stream value. We observe also from Fig. 3 that the velocity increases as the free convection parameter Gr, the magnetic Hartmann number M and the Prandtl number Pr are increased, but is not visibly affected by increase in the exponential index ω. This means that the increase in the velocity observed as a result of increase in Gr, M, and Pr would still be observed whether we our working fluid is compressible or incompressible. It is worthy of note here that $\omega = 0$ corresponds to the case of an incompressible fluid. These observations compliment nicely those reported in Bestman [11].

CURVE	t	Pr	M	Gr	R_a	ω
I	0.05	0.7	0.1	5.0	0.5	0.76
II	0.05	0.7	0.1	5.0	0.5	5.0
III	0.05	0.7	0.1	10	0.5	0.76
IV	0.05	0.7	0.2	5.0	0.5	0.76
V	0.05	0.7	0.1	5.0	0.1	0.76
VI	0.05	7.0	0.1	5.0	0.5	0.76
VII	0.15	0.7	0.1	5.0	0.5	0.76

Fig. 4 shows plots of the skin friction at the plate as a function of the exponential index ω from where we observe that as the Prandtl number increases from, say, $Pr = 0.71$ as corresponds to air, to $Pr = 7.0$, as would correspond to water there is a corresponding increase in the skin friction. The gradient of the curves in Fig. 4 increase as the Prandtl number increases and for $Pr = 0.7$ (not shown), the gradient is zero. In other words the gradient of the curves of the skin friction at the plate increase as you go from a compressible fluid, such as air ($Pr = 0.71$) to an incompressible fluid such as water ($Pr = 7.0$).

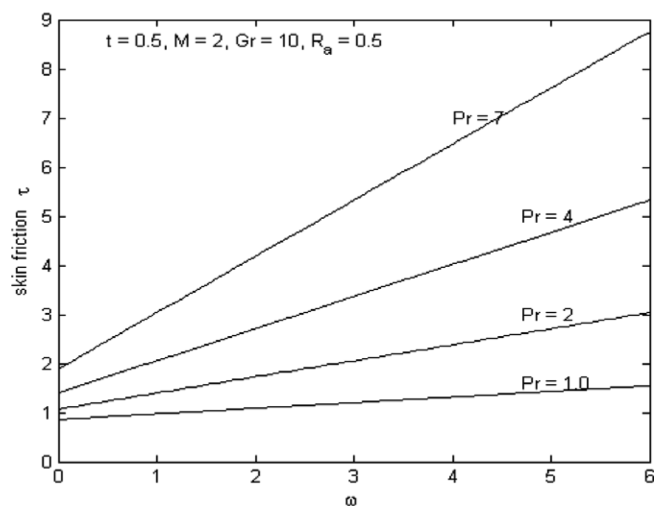

Fig. 4. Skin friction at the wall for a single pulse.

5. Conclusions

We can conclude from our results that for the special case of a single pulse:

- the temperature boundary layer increases as the radiation parameter and the time period are increased,
- increase in the Prandtl number is accompanied by a decrease in the temperature,
- variation of the exponential index ω has little effect on the temperature or the velocity distributions,
- the velocity increases as Gr, Pr, and M are increased, and
- the skin friction for a compressible fluid such as air $Pr = 0.71$, is lower than the skin friction for an incompressible fluid such as water $Pr = 7$.

We accept that there are other methods of solution of problems of this nature but we think, as a first approximation, errors associated with the assumption of Boussinesq approximation for the compressible fluids, are more than made up for by the simplicity of the present model.

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